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Townsend energy factor for slow electrons in dry air

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The Townsend energy factor K_T which is the ratio of mean energy of agitation of the electrons to the mean molecular energy is measured in dry air over the range $3\,06\times10^{-17} \leqslant E/N < 76\,5\times10^{-17}$ V cm^2 (E is the electric field strength in V cm^{-1}, and N the gas density in cm^{-3}). In dry air Huxley's solution of the diffusion equation for electron attaching gases yields results which agree well with those obtained by using the diffusion equation for non-attaching gases. Factors influencing the accurate determination of K_T have also been briefly discussed.

Introdfction

For an electron non-attaching gas the steady state diffusion equation for electrons drifting in a uniform electric field E (parallel to the Z axis) is

$$\nabla^2 n = -\frac{\mu E}{D}\frac{\partial n}{\partial z} \qquad \ldots (1)$$

where n is the electron density, μ the mobility and D the diffusion coefficient. The solution of equation (1) is arrived at by considering the proper boundary conditions which are, that the emission of electrons is from a point source and that the electron density is zero everywhere, *i.e* , the collector plate, the walls and the cathode, except at the source where it is finite. Two different solutions are given for equation (1) by Townsend (1948) and Huxley & Bennett (1940). Townsend's solution neglects the boundary conditions at the anode while that by Huxley & Bennett neglects cathode boundary conditions. With reference to figure 1, if i is the current falling on the central disc of radius a and i is the total current arriving at the receiving electrode, then, Huxley & Bennetts' solution may be written as

$$R = -\frac{i_a}{i} = 1-\frac{h}{d}\exp\left[-\frac{\mu E}{2D}(d_a-h)\right], \qquad \ldots (2)$$

where h is the distance between the cathode and anode and $d_a^2 = h^2+a^2$. Equation (2) assumes that the annulus extends upto infinity, but, in practice, the radius of the annulus c has a finite value in which case the solution is modified to

$$R = \frac{i_a}{i_a+i_c} = \frac{\left\{1-\frac{h}{d_a}\exp\left[-\frac{\mu E}{2D}(d_a-h)\right]\right\}}{\left\{1-\frac{h}{d_c}\exp\left[-\frac{\mu E}{2D}(d_c-h)\right]\right\}}. \qquad \ldots (3)$$

where $d^2 = h^2+c^2$. In figure 1 the central disc is surrounded by two annulii and equation (3) is applicable when the inner and outer annulii are connected together and considered as one, although equation (3) can easily be extended to a multiple ring collector electrode. Therefore the measurement of R directly yields the ratio D/μ when the dimensions of the diffusion apparatus are known.

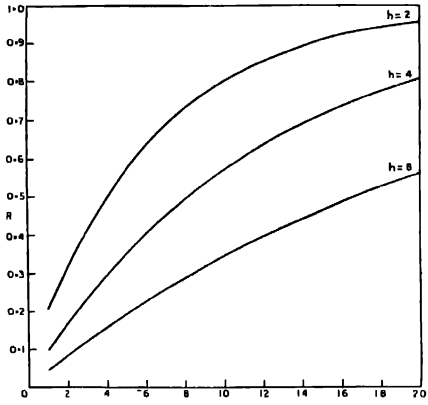

Figure 1 Geometry of the diffusion gap.

In the presence of ionization and electron attachment the steady state diffusion equation for electrons is (neglecting the detachment of electrons from negative ions and recombination of ions)

$$\nabla^2 n = \frac{\mu E}{D}\left[\frac{\partial n}{\partial z} - (\alpha-\eta)n\right] \qquad \ldots (4)$$

where α is the Townsend's primary ionization coefficient, and η is the attachment coefficient. The solution of equation (4) has been worked out by Huxley (1959) and independently by Lucas (1965).

Huxley has shown that the ratio of currents to the annulus $a \leqslant \rho \leqslant b$ to the portion of electrode $a \leqslant \rho \leqslant \infty$ is (a, large enough to eliminate the current due to negative ions)

$$R = \frac{i_{ab}+I_{ab}}{i_{a\infty}+I_{a\infty}} \qquad (5)$$

where electron current and I negative ion current

$$= 1 - \frac{i_{b\infty} + I_{b\infty}}{i_{a\infty} + I_{a\infty}}$$

$$= 1 - \frac{\frac{h}{d_b} \exp\left[\lambda h - u d_b\right] + \frac{\lambda h \eta}{u} \int_0^1 \exp(\lambda h s)\, B\, ds}{\frac{h}{d_a} \exp\left[\lambda h - u d_a\right] + \frac{\lambda h \eta}{u} \int_0^1 \exp(\lambda h s)\, A\, ds} \qquad \ldots (6)$$

where $$B = \exp\left[-uh\left(\frac{b^2}{h^2} + s^2\right)^{\frac{1}{2}}\right] - \exp\left[-uh\left\{\frac{b^2}{h^2} + (2-s)^2\right\}^{\frac{1}{2}}\right]$$

$$A = \exp\left[-uh\left(\frac{a^2}{h^2} + s^2\right)^{\frac{1}{2}}\right] - \exp\left[-uh\left\{\frac{a^2}{h^2} + (2-s)^2\right\}^{\frac{1}{2}}\right]$$

$$d_b^2 = h^2 + b^2;\ 2\lambda = \frac{\text{drift velocity of electrons}}{\text{diffusion coefficient of electrons}} = \frac{W}{D} = \frac{E}{D/\mu};$$

$$u^2 = \lambda^2 + 2\lambda(\eta - \alpha).$$

Thus if a set of curves is drawn for an apparatus showing R as a function of u and η for given value of h, when u and η may be simultaneously found by measuring R at two values of h.

2. Programme For Calculation of R Using Equation (6)

For a diffusion length h and collector electrode dimensions a and b the current ratios R are computed for various constant values of u varying η over a wide range A survey of the published data on D/μ and attachment coefficient η for attaching gases, has enabled us to fix the range of values of u and η for calculation of R using equation (6). First u is kept constant at $u = 1$ and η varied from 0 to 1 at 0.05 intervals and R calculated. This is repeated for values of u varying from 2 to 10. In the second set of calculations for various constant values of u ranging from 0.5 to 10, η is varied from 0 to 0.05 at 0 005 intervals and R calculated. Similarly in a third set of calculations u ranging from 1 to 20, η is varied from 0 to 5 at 0.5 intervals and R calculated Thus 3 sets of curves showing the values of R as a function of (u, η) for $h = 2$, 4 and 8 cm are prepared. The computer programme is written in autocode and has been run on an Elliot 803 model 2 computer The integrand is segmented into 20 segments between 0 and 1.

In a moderately attaching gas such as dry air we can neglect η (see section 4) and the ratio of currents to inner annulus to the total current to both the annuli reduces from equation (6) to

$$R = 1 - \frac{d_a}{d_b} \exp\left[u(d_a - d_b)\right]. \qquad (7)$$

Typical curves showing the variation of R as a function of u at $h = 2$, 4 and 8 cm are given in figure 2. The above equation enables us to measure $u(=\lambda)$ in dry air.

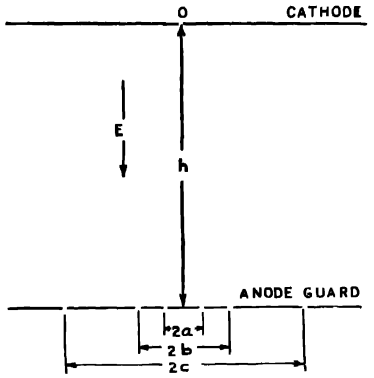

Figure 2. Curve showing current ratio R as a function of u neglecting attachment.

From $2u = \frac{W}{D} = \frac{38\,92E}{K_1}$ (at 25°C), first K_1 = average electron energy factor is evaluated. Then the Townsend energy factor

$$K_T = \frac{\text{mean energy of agitation of the electrons}}{\text{mean energy of the gas molecules}}$$

is obtained by the relation $K = K_1/A$, where A is a constant depending on the energy distribution function of the electrons in the swarm.

$A = 1$ for Maxwellian Law
$= 1.141$ for Druyvesteyn Law.

3. Apparatus and Procedure

The diffusion apparatus is shown schematically in figure 3. The details of the vacuum system and pressure measurement are given in an earlier paper by Raja Rao & Govinda Raju (1971). Nitrogen (cylinder grade N_2 consisting of 99 9 percent N_2, 0.05 percent O_2 and CO_2 5 p.p.m by volume), is passed over copper filings maintained at 400°C and then passed over two liquid air traps to remove traces of CO_2 and water vapour respectively. Similarly air is also passed through the two traps to remove moisture.

The electric field in the diffusion gap is maintained uniform by means of 5 'thick' guard electrodes to which are applied appropriate potentials from a well

regulated power supply through a high stability potential divider chain. These potentials are adjusted using a standard cell and a high sensitivity galvanometer. The stability of the field voltage supply is better than 2 parts in 10^6 and the voltage can be measured to an accuracy better than ± 0.5 percent.

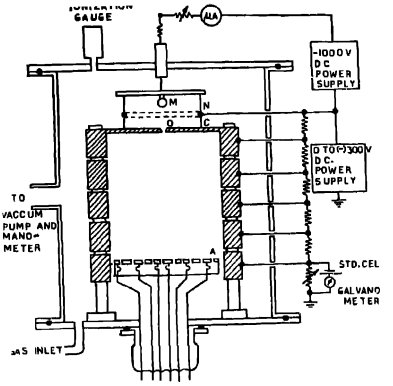

Figure 3 Schematic diagram of the electrode system of the diffusion apparatus.

The guard electrodes are mounted on three quartz rods fixed on a metal ring with P.T.F.E. spacers (1.67 mm thick) between them. The collector electrode is mounted on a 1/2 in. thick P.T F.E. base. It consists of a central disc of nominal diameter 10 mm and four annulii of nominal (outer) diameters of 20, 30, 64 and 85 mm with an air gap of 0.25 mm between them. The anulii are surrounded by a guard ring with an outer diameter of 97 mm. All the rings are made of brass and heavily gold plated to reduce the contact potential differences (from about 200 mV to 40 mV). It is observed that the contact potentials varied with time and after a warm up period of about an hour came down to about 1 or 2 mV. The leakage resistance between adjacent rings and between each ring and earth is greater than $10^{14}\Omega$. The diffusion gap length is variable over a range of 0 to 9 cm by moving the collector plate (anode) axially through a Wilson seal and is measured with a micrometer to an accuracy of ± 0.001 cm. The insulator lead throughs for the electrical connections to the collector rings are made of P.T.F.E.

The source of electrons is a glow discharge between a sphere (M) and a perforated plate (N), both made of brass. The glow discharge current can be varied smoothly and it is less than $200\mu A$ for all the pressures used. To prevent electrons from entering the diffusion gap from the sides, the glow is contained in a pyrex glass cylinder. To avoid photons liberated from the glow discharge reaching the diffusion region the plate N is made of two metal plates with non-aligned

perforations so that the photons are almost prevented from reaching the anode or walls of the diffusion chamber through the source hole. The use of glow discharge for a source of electrons seldom has the disadvantage that the gas may become heated during the course of measurements. In order to keep errors due to temperature changes to a minimum throughout the experiments, the glow discharge is switched on just before the ratio of currents is taken.

The stream of electrons produced by the glow discharge pass into the space between N and C where they acquire a steady state of motion in the same electric field E as that prevailing between C and A (in the diffusion gap) Some of the electrons enter the diffusion gap through a small hole O (1 mm diameter) in the cathode and move through the gap under uniform electric field E to the anode

The experimental parameters are so chosen that all negative ions entering the source hole are collected by the central disc. In order to render negligible the effects of Coulomb repulsion, the total electron current entering the diffusion gap is restricted to about 2×10^{-12} A. These currents are measured with an E.I.L. 33B Vibron electrometer indicator unit used in conjunction with a A 33B converter unit. The absolute accuracy of the current measurements is about ±1.0 percent.

The measurements are made in the pressure range 2-10 torr and for electric fields greater than 8 V/cm.

4. Factors Influencing the Accurate Determination

Factors influencing the accurate determination of K_T or D/μ in an attaching gas are briefly discussed in this section Detailed discussion for non-attaching gases has been given by Crompton & Jory (1962) and Crompton *et al* (1965).

In equation (5) the denominator shows that the total current is the current collected by an annulus extending to infinity, but, in practice, the outer radius c of the outer annulus is finite, and hence the ratio of ratios corresponding to equation (3) in non-attaching case should be used. In this investigation the ratio c/h is found to lie between 0.5 to 2.18 for the diffusion gap lengths used and the error that may creep in due to this approximation is observed to be negligibly small. This is consistent with Crompton *et al* (1965) who observed in almost a similar diffusion apparatus, for the same electrode geometry, with the most widely divergent electron stream, less than 0.004 per cent of the electron swarm lies outside a radial distance of 4 cm.

An analysis of the field produced by guard electrode system is made using the equation (Crompton *et al* 1965).

$$\phi = \frac{Vz}{l} + \sum_N \frac{2V}{N\pi} \frac{I_0(N\pi\, r/l)}{I_0(N\pi\, c/l)} \sin(N\pi z/l) \frac{\sin(N\pi\, g/2l)}{N\pi\, g/2l} \quad \ldots (8)$$

where substituting $l = 10/6$ cm, $c = 5$ cm, $h = 8.17$ cm and $g = 0.167$ cm, ϕ, the potential at any point within an axial cylindrical volume of 10 cm in diameter is found within ± 0.18 per cent of the value corresponding to a uniform field.

The influence of non-uniform surface potentials over the receiving electrode appears to be the factor most likely to limit the accuracy at small values of the parameter E/N. Despite the use of gold plated metal surfaces throughout the collector electrode to minimise effects of this kind, there is evidence to suggest that under certain experimental conditions, the contact potential over the surface of the anode may have been neither uniform nor constant. This results in the ratio changing slightly after a few minutes when the ratio is read at next higher E/N and previous E/N repeated. Because of this effect some possibility exists that the results recorded at the lowest field strengths may be in error. This possibility is reduced by commencing every series of results for a given pressure at the lowest field strength.

Pressures less than 2 torr are not used and the maximum error in the measurement of pressure at 2 torr is less than 2 per cent with further reduces for higher pressures used As mentioned earlier the accuracy to which the field strength is measured is ± 0.5 per cent.

Crompton *et al* (1965) suggest that the lowest value of the field strength should be 3V/cm in order to keep the errors from contact potential differences within acceptable limits but in this study field strengths less than 8V/cm are not used.

It is observed that, in the curve showing R as a function of u neglecting attachment ($\eta = 0$) an error of ± 1 per cent in the measurement of R causes an error of ± 2.7 per cent in u for $h = 2$ cm if $R < 0.8$; an error of ± 1.7 per cent in u for $h = 4$ cm if $R < 0.5$; and an error of ± 1 per cent in u for $h = 8$ cm if $R < 0.35$. But preliminary experiments show that for $R < 0.8$ for $h = 2$ cm, $R < 0.6$ for for $h = 4$ cm and $R < 0.5$ for $h = 8$ cm yield more consistent values of D/μ or K_T and closer in agreement with the results obtained by previous worker.

The ratio of currents given by equation (6) is computed by us for $h = 2$, 4, and 8 cm and for various values of u and η and the ratios show that in dry air the attachment coefficient η/N cannot be evaluated accurately. For example, at any value of E/N say $E/N = 61.3 \times 10^{-17}$V cm^2 ($E/p = 20$), where η/N is maximum in dry air and equal to 11.95×10^{-20} cm^2 ($\eta/p = 0.0039$), $D/\mu = 1.5$. The computed ratio of currents for $h = 4$ is 0.432. The ratio of currents usually is measured to an accuracy of about 0.5 per cent. An uncertainty of ± 0.5 per cent in this will give a value of $\eta/N = 24.5 \times 10^{-20}$ cm^2 and less than 3.06×10^{-20} cm^2 ($\eta/p = 0.008$ and less than 0.001) which mean an error of over cent per cent; this error will increase further at lower values of E/N. But the error caused in evaluation of D/μ or K_T with the same uncertainty in ratio R in less than ± 2.5 per

cent. Hence using equation (7), only values of u and hence K_T for both Maxwellian and Druyvesteyn laws of the distribution of the velocities are determined. Here also K_T is observed to depend on E/N but not on N.

5. Results

To check the reliability of the diffusion apparatus, experiments are carried out in nitrogen. The results obtained for K_T shown in figure 4 generally agreed with those of Crompton & Sutton (1952) within 5 per cent provided appropriate experimental parameters are chosen. In agreement with Crompton & Sutton the values of K_T are observed to be independent of N at any value of E/N in nitrogen.

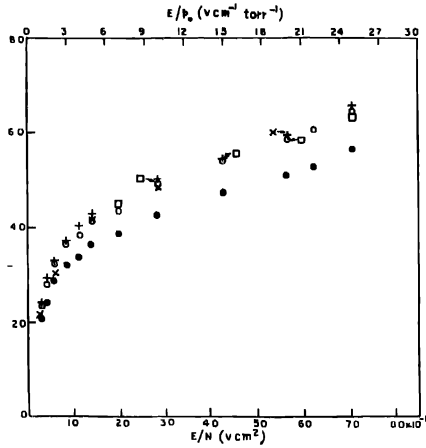

Figure 4. Values of Townsend Energy Factor K_T in nitrogen as a function of E/N.
Present work : o Maxwell o Druyvesteyn + Crompton & Sutton (1951) × Townsend & Bailey (1921) □ Naidu & Prasad (1968)

The results obtained for K_T in a moderately attaching gas such as dry air shown in figure 5 along with those of previous workers. The results in dry air also agreed with those of Crompton *et al* within 5 per cent and no pressure dependence was observed. These values together with the results of previous workers, are shown in figure 5 However, it was observed that consistent values were obtained only if the ratio h/p was kept between 0.4 and 1. The exact significance of this criterion is not clear. We believe that the various factors which restrict the use of equations (6) and (7) (*e g.*, the current due to negative ions entering through the source hole must fall on the central disc and the spread of the electrons

differs greatly from that of ions) become increasingly significant when the value of h/p falls outside this range.

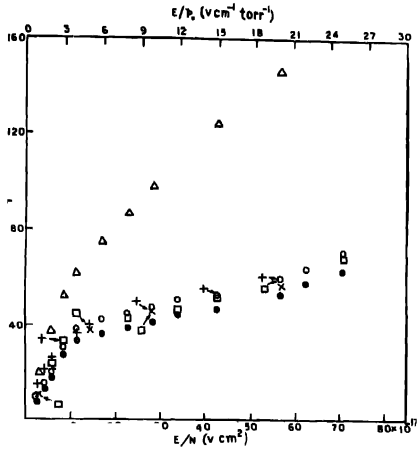

Figure 5 Values of Townsend Energy Factor K in dry air as a function of E/N.
Present work : ● Mawell ● Druyvesteyn
× Townsend & Tizard (1913)
Δ Huxley & Zaazou (1949)
+ Crompton *et al* (1953)
□ Res & Hory (1964)

Table 1. Values of D/μ at $N = 32\,65\times10^{16}$ cm^{-3}

$E/N\times10^{17}$(V cm^2)	Eq (2)		Eq. (9)		Eq (7)	
	(a)$^+$	(b)$^{++}$	(a)	(b)	(a)	(b)
3.065	0 23	0.20	0.35	0 26	0.24	0.23
6 13	0.51	0.50	0.39	0.33	0.52	0 52
9.2	0 80	0 78	0.41	0 34	0.80	0.75

(a)$^+$ D/μ values at $h = 4$ cm, (b)$^{++}$ D/μ values at $h = 8$ cm

The ratio D/μ was observed to be somewhat dependent on h and the magnitude of this effect is shown in table 1 The values obtained at lower spacings were generally larger. From a consideration of spatial dependence of the energy distribution, Parker (1963) has deduced that differences of up to 20 percent could be observed between D/μ values at various diffusion lengths. In a recent paper, Francey (1969) has rederived the spatial dependence of the energy distribution assuming a constant collision cross-section and deduced that the ratio of currents is

$$R = 1-\left\{1-\frac{Dmab^2}{2(kT\mu E-D)Bh^2}\right\}\exp\left\{\frac{-2^{3/2}\mu E}{6B^{\frac{1}{2}}D}\right\}\frac{b^2}{h}, \qquad (9)$$

where m is the mass of electron, $a = (e/m)E$ (e = charge on an electron), k is Boltzman's constant, T is the gas temperature and

$$B = \frac{ma^2ML^2}{6(kT)^2}$$

M being the mass of a gas molecule and L its mean free path. By substituting $m = 9.1091 \times 10^{-31}$ kg, $a = 1.7588 \times 10^{11} E$ ms^{-2}, $M = 46.5 \times 10^{-27}$ kg, $L = 4\sqrt{2} \times L_{atom} = 35.5 \times 10^{-9}$m and $kT = 3.98 \times 10^{-21} J$, we get $B^{\frac{1}{2}} = 1.313 \times 10^{-3} E$ (E in Vm^{-1}).

Using the measured ratio of currents, D/μ values have been calculated using the equations (2), (9) and (7) and compared in table 1. Though Francey's solution predicts that D/μ values decrease with increasing diffusion length at the same value of E/N, the observed difference is smaller possibly because the assumptions made by Francey are inapplicable at higher values of E/N.

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